UNSTEADY HEAT EXCHANGE CLOSE TO A STAGNATION POINT

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An examination is made of the conjugate problem of the heat exchange between a flow and a blunt body in the vicinity of the stagnation point under steady and unsteady conditions.

Several authors [1-3] have theoretically investigated unsteady heat exchange in the vicinity of the stagnation point in fluid flow past a blunt body. The authors of [1], which presents a brief survey of their past studies, examined the case of unsteady heat exchange caused by a stepped change in heat flux and the temperature of the surface of the body. It was assumed that the body was enveloped by a subsonic steady laminar flow of an incompressible fluid with constant properties and that energy dissipation in the flow was negligibly small. The term in the energy equation accounting for convective heat transfer along the surface was omitted on the grounds that the derivative of temperature with respect to the coordinate along the surface was equal to zero at the stagnation point. A solution was found by means of the Laplace transform for small and large intervals of time in the form of series. As an example of the findings, it was shown that, after a stepped change in the wall temperature, heat flux exceeds the corresponding steady-state heat flux. The flux ratio decreases with time, approaching unity, and increases with an increase in the Prandtl number. The relaxation time relative to the heat flux increases in proportion to $Pr^{1/4}$ and changes in inverse proportion to the velocity of the unperturbed flow.

Using the assumptions noted above, the authors of [2] examined the problem of the temperature field in the boundary layer in the vicinity of the stagnation point with a change in the temperature of the incoming flow. With a stepped change in the temperature of the flow at a certain distance from the surface and a surface temperature of zero, investigators evaluated the relaxation time of the boundary layer for three values of the Pr number from 0.5 to 2. It turned out that the relaxation time increased with an increase in the Pr number by a factor somewhat greater than unity.

The authors of [3] investigated unsteady heat exchange after a stepped change in flow temperature for the cases of constant surface temperature and zero heat flux on the rear surface of a thin plate. In the latter case, the temperature was assumed constant through the plate thickness at each moment of time. It was shown that the heat-transfer coefficient decreases with time. An increase in the product of the specific heat and density of the plate material or the plate thickness and a reduction in the thermal conductivity are accompanied by an increase in the transient heat-transfer coefficient α_t and the relaxation time. With an increase in the Pr number, both α_t and the relaxation time decrease.

The authors of the above theoretical works, in studying unsteady heat exchange at an interface, assigned boundary conditions on this surface which necessarily limited the applicability of the results to the imposed conditions. Certain of the conclusions reached in the different studies are mutually inconsistent. One of two variants examined in [3] entailed an approximate accounting of the wall-heating process, which made it possible to investigate the dependence of the heat-transfer characteristics on the properties and dimensions of the body.

It is often the case that the surface temperature, which changes during the heat-exchange process, is commensurate with the temperature of the incoming flow. In this case, a mathematical model which considers the process of heat conduction in the body should describe the processes actually occurring more accurately than the models examined above.

The conjugate transient problem of heat transfer close to the stagnation point of a spherically blunted body in a steady flow may be described as follows with allowance for the assumptions made in [1-3] and noted above: the equations of:

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continuity

$$\frac{\partial (xu)}{\partial x} + \frac{\partial (xv)}{\partial y} = 0, \tag{1}$$

momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

flow energy

$$\frac{\partial t}{\partial \tau_*} + u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} = a_f\frac{\partial^2 t}{\partial y^2}, \tag{3}$$

body heat conductivity

$$\frac{\partial t}{\partial \tau_*} = a_b\left(\frac{\partial^2 t}{\partial y^2}\right), \tag{4}$$

the initial and boundary conditions

$$t(y,\ 0) = t_0, \tag{5}$$

$$t(y_e,\ \tau_*) = t_e, \tag{6}$$

$$t(y_{-\infty},\ \tau_*) = t_0, \tag{7}$$

$$t_f(0,\ \tau_*) = t_b(0,\ \tau_*),\quad \lambda_f\frac{\partial t_f(0,\ \tau_*)}{\partial y} = \lambda_b\frac{\partial t_b(0,\ \tau_*)}{\partial y}, \tag{8}$$

$$u = v = 0 \text{ at } y = 0,\ u = u_e \text{ at } y = y_e, \tag{9}$$

where u, v are the velocity components along the axes x and y; τ_*, dimensional time; $-(1/\rho)\cdot(\partial p/\partial x) = \beta^2 x$; β, velocity gradient, and is related to the velocity u_e by the relation $u_e = \beta x$. For a sphere, $\beta = 3U_\infty/2R$, $x \approx r$ close to the stagnation point. The parameters with the index b pertain to the body, while parameters with the index f pertain to the fluid. The index e denotes correspondence to the outer boundary of the boundary layer. The mathematical model is two-dimensional for the flow and one-dimensional for the body. The choice of such a model was dictated by the fact that the sensors normally used to study transient heat exchange are metal rods insulated on their sides to ensure uniform transfer of heat to the rods. As shown by experimental and theoretical checks [4, 5], this condition is usually met.

Using the dimensionless variables [6]

$$\eta = \left(\frac{\beta}{\nu}\right)^{1/2} y,\quad \Theta = (t - t_0)/(t_e - t_0),\quad \tau = \beta\tau_*/\mathrm{Pr}, \tag{10}$$

and the current function $\psi = (\beta\nu)^{1/2}x^2 f(\eta)$, satisfying the continuity equation, the system of equations may be reduced to dimensionless form. We obtain the following equations:

of momentum

$$f'^2 - 2ff'' = 1 + f''', \tag{11}$$

of energy

$$\frac{\partial \Theta}{\partial \tau} = 2\mathrm{Pr}\, f\frac{\partial \Theta}{\partial \eta} + \frac{\partial^2 \Theta}{\partial \eta^2}, \tag{12}$$

of heat conductivity

$$\frac{\partial \Theta}{\partial \tau} = \frac{a_b}{a_f}\left(\frac{\partial^2 \Theta}{\partial \eta^2}\right), \tag{13}$$

for boundary conditions

$$\Theta(\eta,\ 0) = 0, \tag{14}$$

$$\Theta(\eta_e,\ \tau) = 1, \tag{15}$$

$$\Theta(\eta_{-\infty},\ \tau) = 0, \tag{16}$$

$$\Theta_f(0,\ \tau) = \Theta_b(0,\ \tau),\quad \frac{\partial \Theta_f(0,\ \tau)}{\partial \eta} = \frac{\lambda_b}{\lambda_f}\frac{\partial \Theta_b(0,\ \tau)}{\partial \eta}, \tag{17}$$

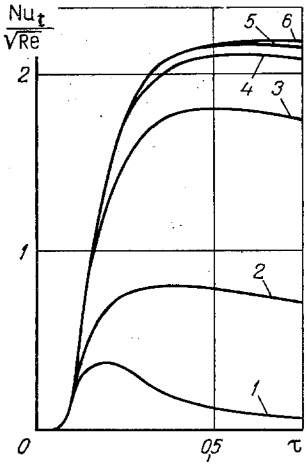

Fig. 1

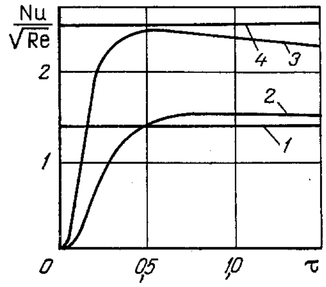

Fig. 2

Fig. 1. Dependence of the complex $Nu_t/\sqrt{Re}$ on dimensionless time (Pr = 5, $k_{c\rho}$ = 0.9): 1) k_λ = 1; 2) 10; 3) 100; 4) 500; 5) 800; 6) 1000.

Fig. 2. Comparison of the complex $Nu/\sqrt{Re}$ under steady (1, 4) and unsteady (2, 3) conditions of heat exchange: 1, 2) Pr = 0.7; 3, 4) 7; 1) k_λ = 10^3–10^4; 2) k_λ = 10^3–$1.5 \cdot 10^4$, $k_{c\rho}$ = $2 \cdot 10^3$; 3) k_λ = 10^3, $k_{c\rho}$ = 0.9; 4) k_λ = 10^3–10^4.

$$f(0) = f'(0) = 0, \quad f'(\eta_e) = 1. \tag{18}$$

In the above formulation, the hydrodynamic part of the problem is independent of the thermal part. The solution to equation (11) with boundary conditions (18) was presented in [7]. It can be approximated with an error no greater than 5% by the relations:

$$f = 0.47\,\eta^{1.8} \quad \text{at} \quad \eta \leqslant 1.4, \tag{19}$$

$$f = \eta - 0.53 \quad \text{at} \quad \eta > 1.4. \tag{20}$$

The solution to the problem (12)-(17) is the temperature fields in the body and the flow, which can be used to calculate the value of the transient criterion Nu_t. The value of Nu_t for a spherical blunting is found from the expression

$$Nu/\sqrt{Re} = \sqrt{3}\,(\partial\Theta_f/\partial\eta)_{\eta=0}\,[1/(1-\Theta_{\eta=0})]. \tag{21}$$

It follows from system (12)-(13) with boundary conditions (14)-(17) and from Eq. (21) that

$$Nu_t/\sqrt{Re} = \varphi(Pr,\ k_\lambda,\ k_{c\rho},\ Sh), \tag{22}$$

where $k_\lambda = \lambda_b/\lambda_f$; $k_{c\rho} = (c\rho)_b/(c\rho)_f$; $Sh = U_\infty\tau^*/D$. The Sh number is related to dimensionless time τ by the relation τ = 3Sh/Pr. Calculations were performed on an ES-1022 computer for values of the parameters Pr = 2-7, k_λ = 1-10^4, $k_{c\rho}$ = 0.3 and 0.6; and Pr = 0.7, k_λ = 10-$1.5 \cdot 10^4$, $k_{c\rho}$ = $2 \cdot 10^3$-$6 \cdot 10^3$.

It was found as a result of the calculations that the complex $Z_t = Nu_t/\sqrt{Re}$ increases relatively rapidly in the initial period of time, then slowing decreases (Fig. 1). The time corresponding to the maximum Z_t lies within the range 0.18 – τ ⩾ 1.5, depending on the Pr number, k_λ, and – to a lesser degree – on $k_{c\rho}$. The maximum shifts in the direction of higher values of time with an increase in k_λ and a decrease in the Pr number. Thus, given constant external parameters of the flow, after a stepped change in its temperature on the outer boundary of the boundary layer, the heat-transfer coefficient changes with time. The increase in α_t in the initial period is due to through heating of the boundary layer, while the reduction in α_t after it reaches its maximum value is due to the effect of the wall on α_t. This effect is connected with the removal of heat inside the body. The value of α_t increases with a relative increase in the thermal conductivity of the wall (with an increase in k_λ), a fact connected with the more intensive heat removal in this case. Meanwhile, the heat-transfer coefficient ceases to be dependent on the parameter k_λ when the latter increases above 800-1000. The processes in the boundary layer become limiting in this case with respect to the mechanism of heat transfer. After through heating of the boundary layer, this limiting case most closely resembles the case of steady-state heat exchange. The effect of the parameter $k_{c\rho}$ on the heat-transfer coefficient depends on the values of the other parameters, but is nonetheless less significant than the effect of k_λ. For example, at Pr = 0.7, a change in $k_{c\rho}$ from $2 \cdot 10^3$ to $6 \cdot 10^3$ does not noticeably change the dependence of Z_t on τ. The dependence of transient heat transfer on the Re number, as would follow from Eq. (22), is the same for the given mathematical model of the process as it is for the steady-state case.

Omitting from system (1)-(4) the terms including time, we obtain a formulation of the conjugate steady-state problem corresponding to the unsteady case examined above. We used the method of counter trial runs [8] to solve the steady-state problem numerically on a "Minsk-32" computer. The complex $Nu_{st}/\sqrt{Re}$ is determined by the parameters k_λ and Pr, i.e., as in the transient case, the complex is a function of the properties of the fluid and the body. In the steady-state case, Z is not dependent on $k_{c\rho}$.

Calculations performed within the range of k_λ $1-10^4$ and Pr 0.01-100 showed that several ranges of change in the parameters may be distinguished, each range being characterized by its own law of change in $Nu/\sqrt{Re}$ in relation to Pr and k_λ. In the first range, ($k_\lambda \cong 10-10^3$, $Pr \leqslant 10^{-1}$), Z_{st} is independent of the Pr number. The value of Z_{st} becomes nearly independent of k_λ as well when the latter rises above 500-1000. Within the indicated range, Z_{st} is determined by the expression

$$Z_{st} = 1.25\,[1 - \exp(-0.772 - 0.0255\,k_\lambda)]. \tag{23}$$

The second range covers values of k_λ from 10 to 10^3 and values of Pr from 10^{-1} to 10^2. The dependence of Z_{st} on k_λ and Pr within this range is satisfactorily described by expressions of the type

$$Z_{st} = a\,[\lg(10^2\,Pr)]^b + c, \tag{24}$$

where the quantity c is equal to the corresponding value of Z_{st} in the first range (23). The coefficients a and b are equal to the following for $k_\lambda \geqslant 20$:

$$a = 0.151 + 0.1\,\{1 - \exp[-2.25(\lg k_\lambda - 1.3)]\}, \tag{25}$$

$$b = 2 + 0.72\,\{1 - \exp[-1.93(\lg k_\lambda - 1.3)]\}. \tag{26}$$

At k_λ greater than 500-1000, Z_{st} is independent of this quantity within this range also. More precisely, the boundary of the region in which Z_{st} is independent of k_λ in the range Pr = 0.01-100 is determined by the expression

$$Z_{bd} = 5.2\lg k_\lambda - 12. \tag{27}$$

The change in Z in the third region (k_λ = 1-10, Pr = 0.01-100) is approximated by the expressions

$$\text{for}\quad Pr \cong 0.01 - 0.5 \quad Z_{st} = 1.62^{\lg k_\lambda} - 0.85, \tag{28}$$

$$\text{for}\quad Pr \cong 0.5 - 100 \quad Z_{st} = (1.71\,Pr^{0.069})^{\lg k_\lambda} - 0.85. \tag{29}$$

The approximation error of Eq. (23) is 6.3%; (24) – 8%; (28), (29) – 3%.

The nonconjugate steady-state heat-transfer problem in the stagnation point region was solved in [6] by Sibulkin. The assumption on the equality of the surface temperature to zero made in [6] is closest to the case of large values of k_λ for the problem as formulated in the present work. The value of $Nu/\sqrt{Re}$ = 1.32 obtained in [6] for the case Pr = 1 agrees satisfactorily with the value of this quantity found from solving the conjugate problem at $k_\lambda > 100$.

Comparison of the solutions obtained for the conjugate problem for steady-state and transient cases showed that, given constant values of the parameters of the unperturbed flow equal to parameters in the steady-state case, in the initial period of the transient heat exchange process the heat-transfer coefficient may differ several times from its steady-state value (Fig. 2). After through heating of the boundary layer, the coefficients are the same in each case.

The following formula can be used to evaluate the time over which the transient heat-transfer coefficient reaches its maximum value (see Fig. 1) at values of Pr = 2-7,

$k_{c\rho} = 0.3\text{-}0.9$, $k_\lambda = 1\text{-}10^3$:

$$\tau_{max} = (0.26 + 0.07\,k_{cp})\,[\exp(-0.16\,\mathrm{Pr})]\,\lg k_\lambda + 0.185, \tag{30}$$

The approximation error of this formula is 1-2% at Pr = 7 and 10% at Pr = 2.

The value of the maximum complex $(Nu_t/\sqrt{Re})_{max}$ is determined with an error of 5% by the expression

$$(Nu_t/\sqrt{Re})_{max} = (0.15\,\mathrm{Pr} + 1.35)\,\{1 - \exp[-(0.0121 + 0.00517\,k_{cp} - 0.00033\,\mathrm{Pr})\,k_\lambda + 0.36 - 0.01\,\mathrm{Pr}]\}. \tag{31}$$

Introduction of the relative quantities $Nu_t/(Nu_t)_{max}$ and $\bar{\tau} = \tau/(\tau_{max} - 0.05/\tau_{max})$ made it possible to combine the theoretical relations into a single relation which takes the following form at $\bar{\tau} \cong 0.08\text{-}1$:

$$Nu_t/(Nu_t)_{max} = 1 - \exp(0.35 - 5.46\,\bar{\tau}). \tag{32}$$

Equations (30)-(32) describe the transient heat-transfer coefficient in the interaction of a water flow with blunt bodies made of metals and insulators in the stagnation-point region for the case of a stepped change in the temperature of the flow beyond the limits of the boundary layer, with the initial flow temperature being equal to the temperature of the body.

The difference for air (Fig. 2) is that here the maximum of the curve of $Nu_t/\sqrt{Re} = f(\tau)$ is not as pronounced as for water. The character of the change in α_t in the air flow on the increasing section is also described by a relation of the type (32) if we take for the value of τ_{max} a value corresponding to, say, $0.98(Nu_t/\sqrt{Re})_{max}$.

In conformity with the notation used in the present work, the dimensional time $\tau^* = \mathrm{Pr}R\tau/3U_\infty$. If we take $R = 10^{-2}$ m — which corresponds to the size of the bodies used in experiments — then for air at $U_\infty = 10^3$ m/sec the time of the change in the heat-transfer coefficient to a maximum value of the same order as the heating time of the boundary layer is $\sim 10^{-6}$ sec. For water at $U_\infty = 1$ m/sec, this time is 10^{-2} sec. Thus, in the case of the flow of a viscous fluid at low velocities, the change in the heat-transfer coefficient resulting from a stepped change in the temperature of the flow outside the boundary layer may be observed by experimental methods described in the literature. In the case of an air flow — including at high temperatures — the change in α_t resulting from the same stepped change in flow temperature beyond the boundary layer may be significant in impulsive processes.

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